## Activity 12 Scalar product (dot product)

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	a)	0	dotP(p,q)
	b)	15	0 dotP(p,r) 15
	c)	24	dotP(q,r) 24
	d)	47	dotP(r,s) 47
	e)	-18	dotP(s,t) -18
	f)	7 <i>a</i> +2 <i>b</i>	dotP(s,u) 7•a+2•b
	g)	ac+bd	dotP(u,v) a·c+b·d
			dotP(u, [2b, -2a])
			Alg Decimal Real Deg 🛄

## 2.

3.

- a) The scalar product of perpendicular vectors is zero.
- b) The scalar product of two vectors in component form is simply the sum of the products of the **i** components and the product of the **j** components, i.e. for vectors  $\mathbf{u}=[a,b]$  and  $\mathbf{v}=[c,d]$  the scalar product  $\mathbf{u} \cdot \mathbf{v} = ac + bd$ .

a) 
$$\mathbf{u} = [m, \angle \alpha]$$
,  $\mathbf{v} = [n, \angle \beta]$ 

b)  $\mathbf{u} = [m \cos \alpha, m \sin \alpha]$  $\mathbf{v} = [n \cos \beta, n \sin \beta]$ 

[m, 2	(α)] <b>⇒</b> u							
		I	[m·cos(	α) m•sir	ı(α)]			
[n, 2	(β)] <b>⇒</b> v					_		
			[n•cos(	β) n•sin	ι(β)]			
dotP (	u, v)							
$m \cdot n \cdot \cos(\alpha) \cdot \cos(\beta) + m \cdot n \cdot \sin(\alpha) \cdot \sin(\alpha)$								
tColle	ct(ans)							
			1	m•n•cos (	α-β)			
þ						•		
Alg	Decimal	Real	Deg		1	(1)		

c)  $\mathbf{u} \cdot \mathbf{v} = mn \cos \alpha \cos \beta + mn \sin \alpha \sin \beta$ 

d) 
$$\mathbf{u} \cdot \mathbf{v} = mn\cos(\alpha - \beta)$$

e)  $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}| |\mathbf{v}| \cos \theta$ 

Where  $\theta$  is the angle between **u** and **v**.

4.

a) 
$$|\mathbf{v} - \mathbf{u}|^2 = |\mathbf{u}|^2 + |\mathbf{v}|^2 - 2|\mathbf{u}||\mathbf{v}|\cos\theta$$

 $\left|\mathbf{u}\right|^2 = a^2 + b^2$ b)  $\left|\mathbf{v}\right|^2 = c^2 + d^2$ and  $|\mathbf{v} - \mathbf{u}|^2 = (c - a)^2 + (d - b)^2$ 

Hence,

$$|\mathbf{v} - \mathbf{u}|^2 = |\mathbf{u}|^2 + |\mathbf{v}|^2 - 2|\mathbf{u}||\mathbf{v}|\cos\theta$$
$$(c-a)^2 + (d-b)^2 = a^2 + b^2 + c^2 + d^2 - 2|\mathbf{u}||\mathbf{v}|\cos\theta$$
$$c^2 - 2ac + a^2 + d^2 - 2bd + b^2 = a^2 + b^2 + c^2 + d^2 - 2|\mathbf{u}||\mathbf{v}|\cos\theta$$
$$-2(ac + bd) = -2|\mathbf{u}||\mathbf{v}|\cos\theta$$
$$ac + bd = |\mathbf{u}||\mathbf{v}|\cos\theta$$

as required.

5.

a) 
$$\mathbf{r}_{f}(t) = [4+4t, -26+12t] \text{ km.}$$

b) 
$$_{f}\mathbf{r}_{p}(t) = \mathbf{r}_{f}(t) - \mathbf{r}_{p}(t)$$
  
=  $[-13+4t, -29+12t]$  km.

c) At closest approach, 
$${}_{\mathbf{f}}\mathbf{r}_{\mathbf{p}}(t)$$
 will be perpendicular to the direction of  $\mathbf{r}_{\mathbf{f}}(t)$ .

This implies that  $_{f}\mathbf{r}_{p}(t) \cdot \mathbf{v}_{f}(t) = 0$ .

So, time of minimum separation is 12:30 p.m. Fishing vessel will be at [14,4] km at this time. Distance between fishing vessel and patrol boat at this time is  $\sqrt{10}$  km.

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0.5 <u>1</u> ₽2	b ∫dx Sim	<u>fdx</u>	<ul> <li>♥</li> <li>♥</li></ul>		Þ
		m•n•e	cos(α·	-β)	3
[4+4	t,-26+12t]	⇒rf		_	
	[-	4•t+4	12•t-	26]	
[17,	3] <b>⇒</b> rp				
			[17	31	
rf-r	⇒frp				
	-	t-13	12•t-	29]	
solve	(dotP(frp,[	4.121	)=0.t	;	
		-,			
			[t=	$\frac{5}{2}$	
rf   ar	IS			_	
			[14	4]	
norm	$(frp t=\frac{5}{2})$			- 1	
norm	2'				
			١	10	
					7
Alg	Standard	Real	Deg	1 .	(III