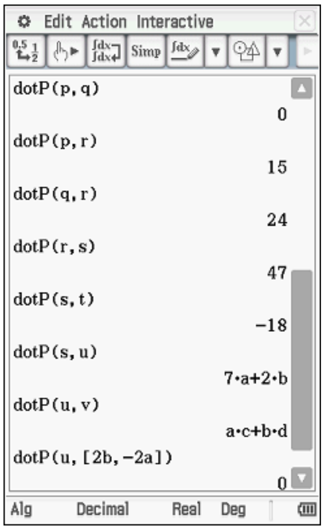
Activity 12 Scalar product (dot product)

1.



a) 0

b) 15

c) 24

d) 47

e) –18

f) $7a+2b$

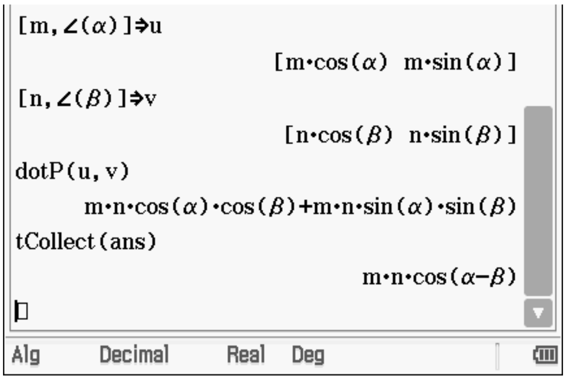
g) ac+bd

2.

a) The scalar product of perpendicular vectors is zero.

b) The scalar product of two vectors in component form is simply the sum of the products of the **i** components and the product of the **j** components, i.e. for vectors **u**=[a,b] and **v**=[c,d] the scalar product $\mathbf{u}\cdot\mathbf{v} = ac + bd$.

3.



a) $\mathbf{u} = [m,\ \angle\alpha]$, $\mathbf{v} = [n,\ \angle\beta]$

b) $\mathbf{u} = [m\cos\alpha,\ m\sin\alpha]$
$\mathbf{v} = [n\cos\beta,\ n\sin\beta]$

c) $\mathbf{u}\cdot\mathbf{v} = mn\cos\alpha\cos\beta + mn\sin\alpha\sin\beta$

d) $\mathbf{u}\cdot\mathbf{v} = mn\cos(\alpha-\beta)$

e) $\mathbf{u}\cdot\mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos\theta$ Where θ is the angle between **u** and **v**.

4.

a) $|\mathbf{v}-\mathbf{u}|^2 = |\mathbf{u}|^2 + |\mathbf{v}|^2 - 2|\mathbf{u}||\mathbf{v}|\cos\theta$

b) $|\mathbf{u}|^2 = a^2 + b^2$

$|\mathbf{v}|^2 = c^2 + d^2$

and $|\mathbf{v} - \mathbf{u}|^2 = (c-a)^2 + (d-b)^2$

Hence,

$$|\mathbf{v} - \mathbf{u}|^2 = |\mathbf{u}|^2 + |\mathbf{v}|^2 - 2|\mathbf{u}||\mathbf{v}|\cos\theta$$
$$(c-a)^2 + (d-b)^2 = a^2 + b^2 + c^2 + d^2 - 2|\mathbf{u}||\mathbf{v}|\cos\theta$$
$$c^2 - 2ac + a^2 + d^2 - 2bd + b^2 = a^2 + b^2 + c^2 + d^2 - 2|\mathbf{u}||\mathbf{v}|\cos\theta$$
$$-2(ac + bd) = -2|\mathbf{u}||\mathbf{v}|\cos\theta$$
$$ac + bd = |\mathbf{u}||\mathbf{v}|\cos\theta$$

as required.

5.

a) $\mathbf{r}_f(t) = [4+4t\ ,\ -26+12t]$ km.

b) ${}_f\mathbf{r}_p(t) = \mathbf{r}_f(t) - \mathbf{r}_p(t)$

$= [-13+4t\ ,\ -29+12t]$ km.

c) At closest approach, ${}_f\mathbf{r}_p(t)$ will be perpendicular to the direction of $\mathbf{r}_f(t)$.

This implies that ${}_f\mathbf{r}_p(t) \cdot \mathbf{v}_f(t) = 0$.

So, time of minimum separation is 12:30 p.m.
Fishing vessel will be at [14,4] km at this time.
Distance between fishing vessel and patrol boat at this time is $\sqrt{10}$ km.